## Girraween High School

# 2024 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Advanced

**General Instructions**

- Reading time: 10 minutes
- Working time: 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Answer multiple-choice questions on the multiple-choice answer sheet provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

**Total Marks: 100**

**Section I 10 Marks**

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

**Section II 90 marks**

- Attempt Questions 11 - 34
- Allow about 2 hours and 45 minutes for this section

## Section I

**10 marks**

**Attempt Questions 1–10**

**Allow about 15 minutes for this section**

**Use the multiple-choice answer sheet for Questions 1–10**

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**Question 1**

What is the gradient of the tangent to the curve $y = \sqrt{10 - x}$ at $x = 1$?

(A) $\frac{1}{6}$

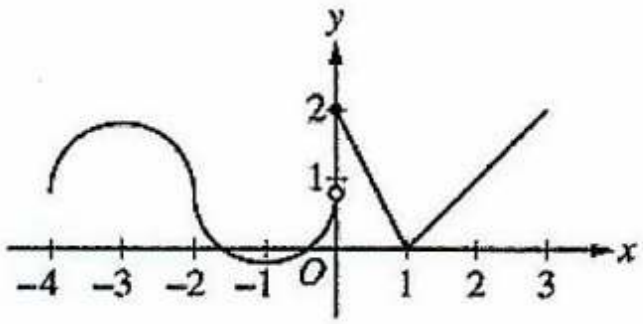
(B) $-\frac{1}{6}$

(C) $6$

(D) $-6$

**Question 2**

The graph of the piecemeal function $y = f(x)$ is shown below. The part of the graph in the domain $-4 \leq x < 0$ comprises two semi-circles, joined at $x = -2$. What are the values $x$ for $-4 < x < 3$, at which the function is continuous but not differentiable?




(A) $x = 1$ only

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

*Examination continues next page…*

**Question 3**

What is the solution of $3^{2x} = 5$ ?

(A) $x = \frac{2 \ln 5}{\ln 3}$

(B) $x = \frac{\ln 5}{2 \ln 3}$

(C) $x = \frac{2 \ln 3}{\ln 5}$

(D) $x = \frac{\ln 3}{2 \ln 5}$

**Question 4**

If $a > 0$ and the function $f(x) = ax^3 + bx^2 + cx + d$ is always increasing, what is the condition for $a$, $b$ and $c$?

(A) $b^2 - 2ac > 0$

(B) $b^2 - 2ac < 0$

(C) $b^2 - 3ac > 0$

(D) $b^2 - 3ac < 0$

**Question 5**

What is the number of solution(s) of the equation $log_e |x^2 - 1| = 0$ ?

(A) 2

(B) 3

(C) 1

(D) 0

***Examination continues next page…***

**Question 6**

Given that $f(x) = \dfrac{4x^5 - 8x}{x^3}$, what is the value of $f'(2)$?

(A) 2

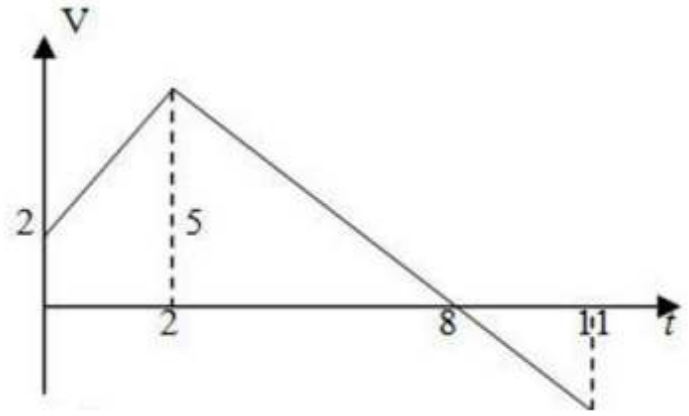
(B) 8

(C) 12

(D) 18

**Question 7**

The diagram gives the velocity of a particle moving in a straight line for 11 seconds.




At $t = 11$, the particle is

(A) 7 metres to the right of the starting point.

(B) 15 metres to the right of the starting point.

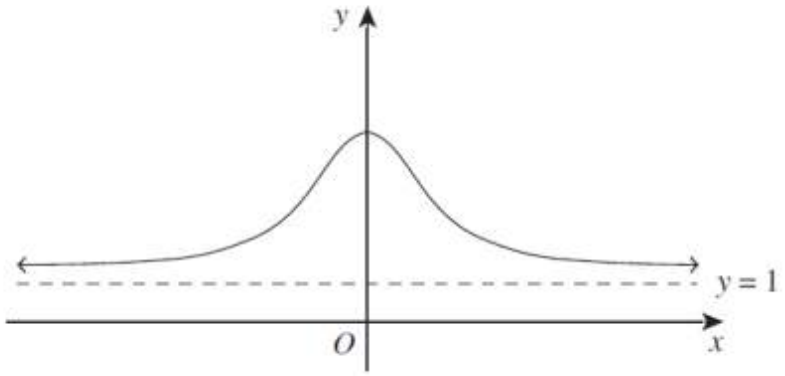
(C) 22 metres to the right of the starting point.

(D) 18.25 metres to the right of the starting point.

***Examination continues next page…***

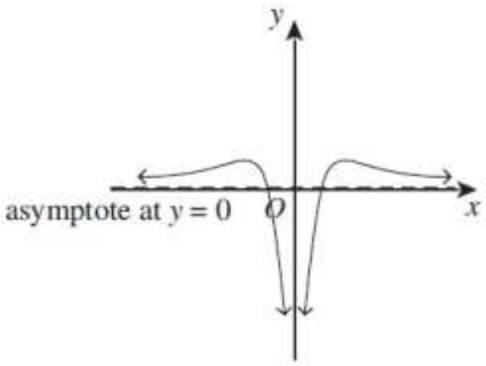
**Question 8**

The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = 1$.

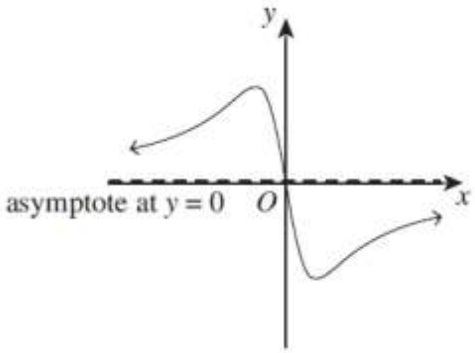



Which of the following could be the graph of $y = f'(x)$?

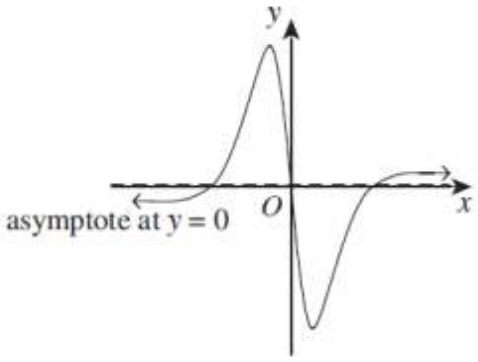
(A)



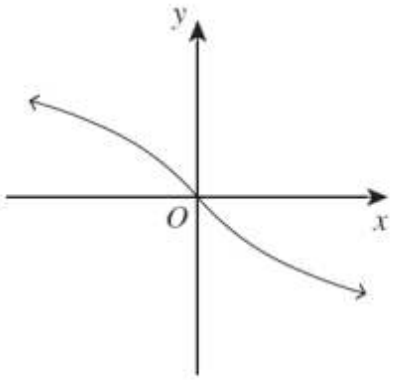

(B)




(C)




(D)

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**Question 9**

Given $\int_2^6 f(x)\ dx = 3$, what is the value of $\int_4^6 f(2(x-3))\ dx$?

(A) $\frac{1}{2}$

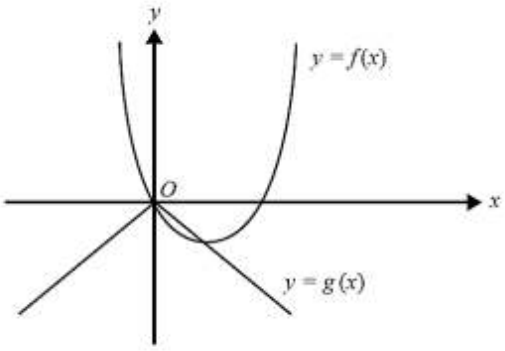
(B) $\frac{3}{2}$

(C) 3

(D) 8

**Question 10**

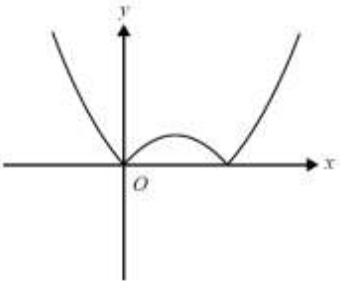
The graph of $y = f(x)$ and $y = g(x)$ are shown below.



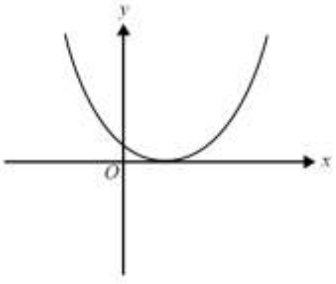

NOT TO SCALE

The graph of $y = f(g(x))$ is best represented by

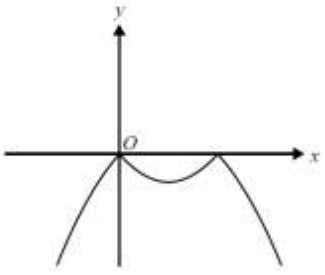
(A)



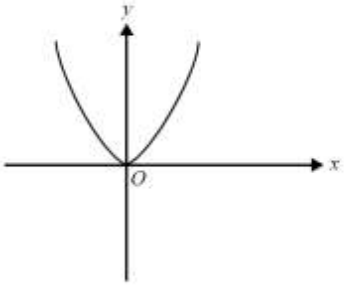

(B)




(C)




(D)

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## Section II

**90 marks**
**Attempt Questions 11–34**
**Allow about 2 hours and 45 minutes for this section**

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

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**Question 11**

Solve $|4 - 2x| = 3$ 2

**Question 12**

Sketch $y = \frac{3}{x+2} - 1$, showing all intercepts with the coordinate axes and all asymptotes. 3

*Examination continues next page…*

**Question 13**

Differentiate with respect to $x$:

a) $y = (3x^2 + 7)^6$ 2

b) $y = 3x^2 \sin x$ 2

**Question 14**

a) $\int \frac{e^{2x}+1}{e^{2x}}\, dx$ 2

***Examination continues next page…***

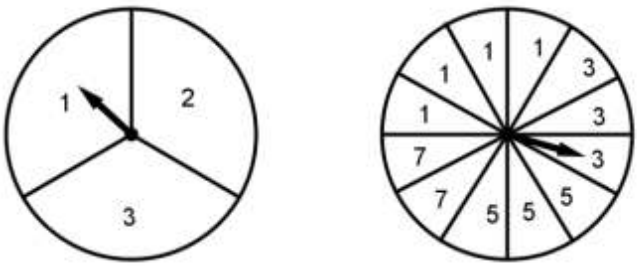
b) Evaluate $\int_0^2 x\sqrt{2x^2+1}\,dx$ 2

*Examination continues next page…*

**Question 15**

The diagram below shows two spinners. 2

1st Spinner 2nd Spinner




Each of the three outcomes on the first spinner are equally likely.

On the second spinner there are 12 equally likely sectors for the arrow to land on with four possible outcomes.

In a game, both spinners are spun simultaneously.

The players score is the sum of the two numbers that the spinners land on.

In the diagram above, the player's score would be 4 since the first spinner landed on 1 and the second spinner landed on 3.

A player wins if their score is an odd number greater than 6.

What is the probability that a player will win on a single turn?

***Examination continues next page…***

**Question 16**

Jim tosses 2 dice with faces numbered 1 to 6. He records the maximum of the two upper faces as a score. The sample space is shown below.

| Dice 1 / Dice 2 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 2 | 3 | 4 | 5 | 6 |
| 3 | 3 | 3 | 3 | 4 | 5 | 6 |
| 4 | 4 | 4 | 4 | 4 | 5 | 6 |
| 5 | 5 | 5 | 5 | 5 | 5 | 6 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

a) Jim plays a game with the 2 dice using the results stated above. 2

He wins \$1 if the score is 3 or 4, \$2 if the score is 6 and \$3 if it is 5.

But he loses \$1 if it is 1 or 2.

Complete the probability distribution table for random variable $Y$ showing the probability of winning and losing on each amount.

| $y$ | -\$1 | \$1 | \$2 | \$3 |
|---|---|---|---|---|
| $P(Y = y)$ | | | | |

b) Find the expected value of $Y$. 2

***Examination continues next page…***

c) If it costs Jim \$2 to play the game, would you expect him to win or lose money? 1

Justify your answer with calculations.

**Question 17**

For a particular curve it is given that $\frac{d^2y}{dx^2} = 12x - 2$. 3

The tangent at $(1, -2)$ on the curve makes an angle at $45^o$ with the positive direction of the $x$ axis.

Find the equation of the curve.

***Examination continues next page…***

**Question 18**

If $f(x) = \sqrt{x}$ and $g(x) = 4 - x^2$, what is the domain of the function $f(g(x))$? 2

**Question 19**

The point A (1, 4) lies on the graph $y = f(x)$. When the function is transformed into the graph $y = -2f(x + 1)$, what are the co-ordinates of the point corresponding to point A? 2

***Examination continues next page…***

**Question 20**

The energy released by an earthquake, $E$, can be given by $\log_{10} E = 11.8 + 1.5M$ where M is the measurement of its magnitude on the Richter scale.

a) Calculate the energy released by an earthquake measuring 6 on the Richter scale. 2
Leave your answer in index form.

b) If there is another earthquake measuring 3.1 on the Richter scale, how many times more energy is released by the first earthquake than by the second? 2
Answer to the nearest whole number.

***Examination continues next page…***

**Question 21**

The closure of several manufacturing businesses in a small town has caused the town's population, $P$, to decline over the last decade. At the start of 2012 the population was 15 000. At the start of 2022 it was estimated to be 9 500.

Assume that the population decline is proportional to $P$, so that $\frac{dP}{dt} = -kP$, where $k$ is a positive constant and $t$ is the time in years.

a) Show that $P = 15000e^{-kt}$ satisfies the differential equation. 1

b) Find the value of $k$ in exact form. 2

*Examination continues next page…*

c) If the population continues to decrease at the same rate, in what calendar year will the population drop to 5 000? 2

*Examination continues next page…*

**Question 22**

A particle starts to move from the origin with a velocity 2 m/s along the $x$-axis.

Its acceleration at any time $t$ seconds is given by $a = \frac{6}{3t+4}$.

a) Show that the velocity, $v$, of the particle at time $t$ is 2

$$v = 2\ln\left(\frac{3}{4}t + 1\right) + 2 \quad m/s$$

***Examination continues next page…***

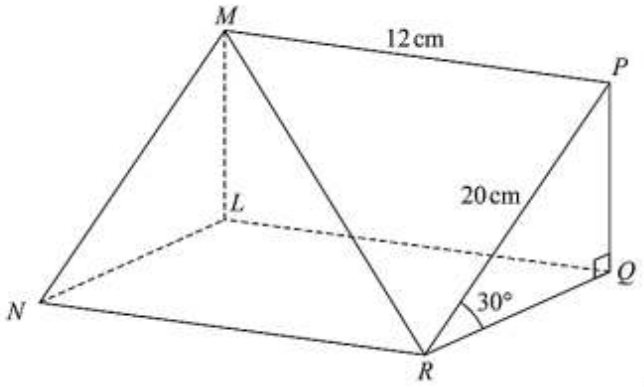
b) Find the time taken by the particle to reach a velocity $4m/s$. 2

c) Does the particle ever return to the origin? Justify your answer. 1

***Examination continues next page…***

**Question 23**

The diagram shows a triangular prism. 3




Not to scale

Triangle PQR is a cross section of the prism.

$PR = 20cm,\ MP = 12cm,\ \angle PRQ = 30^o,\ \angle PQR = 90^o$

Calculate the size of the angle that the line *MR* makes with the plane *RQLN*. Give your answer correct to 1 decimal place.

***Examination continues next page…***

**Question 24**

The third term of a geometric series is 48 and the sixth term is $\frac{81}{4}$.

a) Find the common ratio and the first term. 2

b) Find the sum of the first ten terms to 4 significant figures. 2

***Examination continues next page…***

**Question 25**

Consider the function $y = (\ln x)^2$.

a) Find its stationary point and determine its nature. 3

*Examination continues next page…*

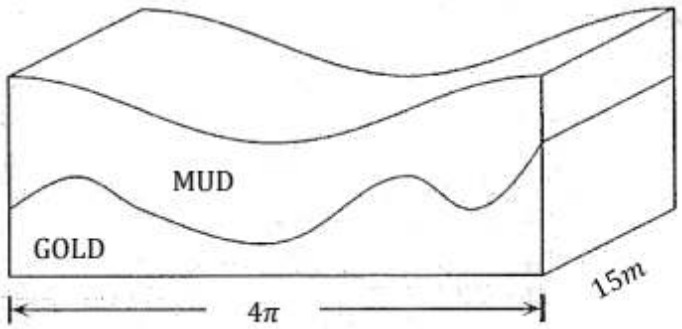
b) Show that there is a point of inflexion at $(e, 1)$. 2

c) Sketch the function showing all essential features. 1

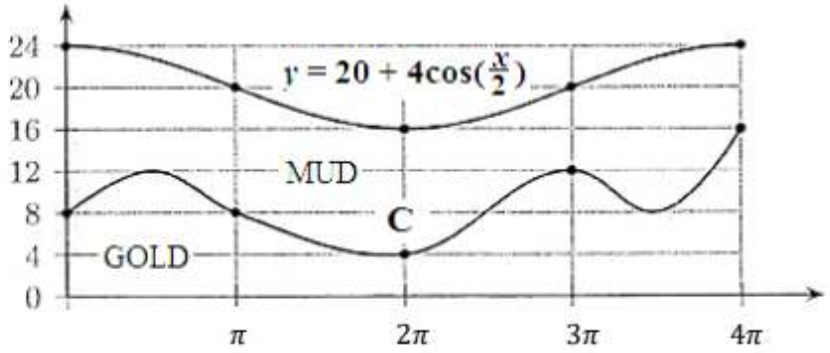
***Examination continues next page…***

**Question 26**

The diagram below shows an amount of gold ore, which is in the shape of a prism underneath a large amount of mud. The width of the prism is $4\pi$ metres and its length is 15 metres.




The graph below shows the cross-section of the prism. The top of the mud is given by the function $y = 20 + 4\cos\frac{x}{2}$ and the top of the gold is shown by the curve $C$.




a) Find, by integration, the total area of the cross-section of the prism, ie: the area of both the mud and gold. 2

b) Using the Trapezoidal Rule with the five function values on the graph, find an estimate for the area of the cross-section of the **gold**. 2

c) Find the volume of the mud. 1

***Examination continues next page…***

**Question 27**

The sum $S_n$ of the first $n$ terms of a certain series is $2n + 3n^2$, for $n \geq 1$. 3

Find an expression for the $n$th term, $T_n$ of the series.

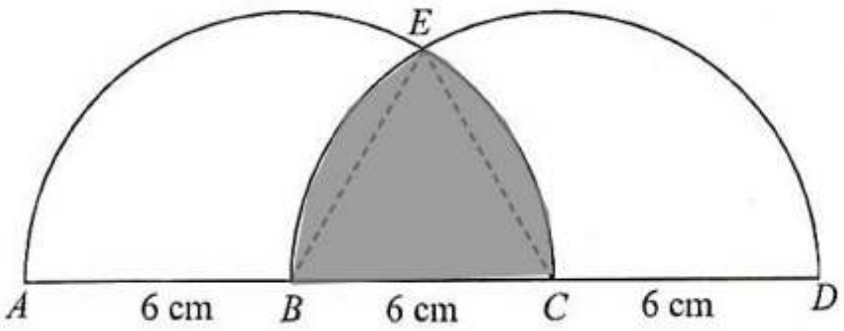
*Examination continues next page…*

**Question 28**

In the diagram, $ABCD$ is a straight line where $AB = BC = CD = 6cm$. 3

The semi-circles on diameter AC and BD intersect at $E$. $\Delta EBC$ is equilateral.

Find in simplest exact form the area of the shaded region common to the two semi-circles.

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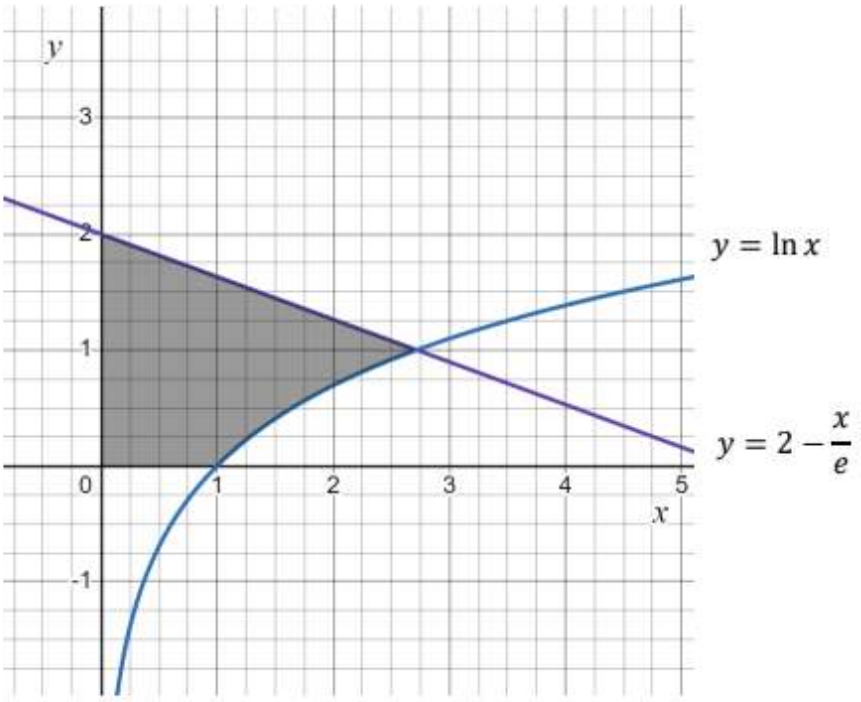
***Examination continues next page…***

**Question 29**

The shaded region below represents the area bounded by the $x$ and $y$ axes, the line $y = 2 - \frac{x}{e}$ and the curve $y = ln\, x$.




a) Show, by substitution, that the line $y = 2 - \frac{x}{e}$ and the curve $y = ln\, x$ intersect at the point $(e, 1)$. 1

***Examination continues next page…***

b) Hence, find the exact area of the shaded region. 3

*Examination continues next page…*

**Question 30**

a) Use the change of base law to show the following is a geometric series and find the common ratio. 2

$$log_3 x + log_9 x + log_{81} x + \cdots$$

b) Hence, show that the sum to infinity of the series 2

$$log_3 x + log_9 x + log_{81} x + \cdots \text{ is } \quad log_3 x^2.$$

***Examination continues next page…***

**Question 31**

If $x = \frac{e^y - e^{-y}}{2}$ use the substitution $m = e^y$ to solve the equation for $y$ in terms of $x$. 4

*Examination continues next page…*

**Question 32**

A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.
The results indicate that:

- 8 students chose to study neither Physics nor Chemistry
- 8 students chose to study Chemistry
- 12 students chose to study Physics.

a) Draw a Venn diagram showing this information. 1

b) Let a student choosing to study Chemistry be event A. Let a student choosing to study Physics be event B. If a student is chosen at random, determine whether events A and B are independent of each other. 2

*Examination continues next page…*

**Question 33**

The following functions represent 3

the number of daytime hours $D = 11.87 + 2.35\cos\frac{\pi}{183}(t + 10)$

and the number of night time hours $N = 12.13 - 2.35\cos\frac{\pi}{183}(t + 10)$

each day in Sydney during 2024, where $t = 0$ is 1st January, 2024.

Find the two values of $t$ when the number of hours of daytime and night time in Sydney are equal.

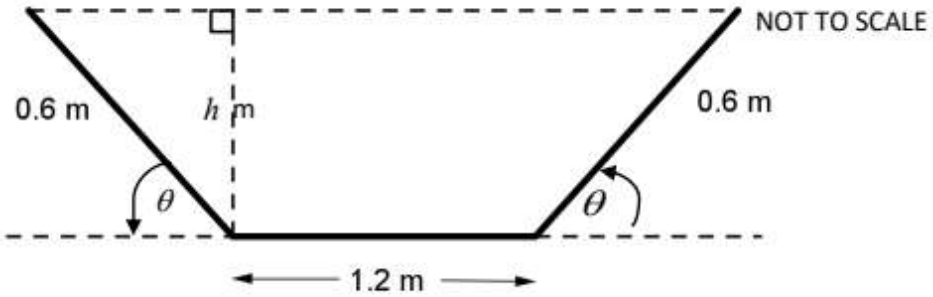
***Examination continues next page…***

**Question 34**

A water overflow channel is to be dug next to a dam. The base of the channel is to be 1.2 metres wide and 15 metres long.

The channel is dug such that the left and right sides are on an angle of $\theta$ with the horizontal. The cross-sectional view of the channel is a trapezium with two sides of length 0.6 m.

The diagram below shows a cross-sectional view of the channel.




a) Find an expression for $h$ in terms of $\theta$. 1

***Examination continues next page…***

b) Show that the volume of the channel is given by 2

$$V = 5.4(2\sin\theta + \sin\theta\cos\theta) \text{ cubic metres}$$

***Examination continues next page…***

c) Find the value of the angle $\theta$ so that the volume of the channel is a **maximum**. 4

Give your answer correct to the nearest minute.

*More working space on the next page…*

***End of examination.***

# Year 12 Trial HSC Examination

## Mathematics Advanced 2024

### Multiple Choice Answer Sheet

**Student Number** Solutions

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. | A | | B | ● | C | ○ | D | ○ |
| 2. | A | ○ | B | ○ | C | ● | D | ○ |
| 3. | A | ○ | B | ● | C | ○ | D | ○ |
| 4. | A | ○ | B | ○ | C | ○ | D | ● |
| 5. | A | ○ | B | ● | C | ○ | D | ○ |
| 6. | A | ○ | B | ○ | C | ○ | D | ● |
| 7. | A | ○ | B | ○ | C | ○ | D | ● |
| 8. | A | ○ | B | ● | C | ○ | D | ○ |
| 9. | A | ○ | B | ● | C | ○ | D | ○ |
| 10 | A | ○ | B | ○ | C | ○ | D | ● |

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## Section I

**10 marks**

**Attempt Questions 1–10**

**Allow about 15 minutes for this section**

**Use the multiple-choice answer sheet for Questions 1–10**

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**Question 1**

What is the gradient of the tangent to the curve $y = \sqrt{10 - x}$ at $x = 1$?

(A) $\frac{1}{6}$

(B) $-\frac{1}{6}$

(C) 6

(D) $-6$

$y = (10-x)^{\frac{1}{2}}$

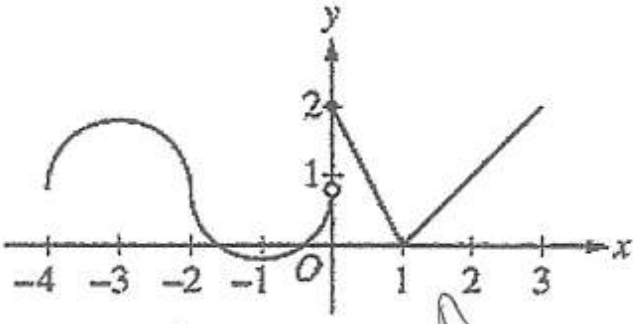
$\frac{dy}{dx} = \frac{1}{2}(10-x)^{-\frac{1}{2}} \times -1$

at $x=1$ $m = -\frac{1}{2}(10-1)^{-\frac{1}{2}}$

$= -\frac{1}{2} \cdot \frac{1}{3} = -\frac{1}{6}$

**Question 2**

The graph of the piecemeal function $y = f(x)$ is shown below. The part of the graph in the domain $-4 \leq x < 0$ comprises two semi-circles, joined at $x = -2$. What are the values $x$ for $-4 < x < 3$, at which the function is continuous but not differentiable?




point

vertical tangent

(A) $x = 1$ only

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

*Examination continues next page…*

**Question 3**

What is the solution of $3^{2x} = 5$ ?

(A) $x = \frac{2\ln 5}{\ln 3}$

(B) $x = \frac{\ln 5}{2\ln 3}$

(C) $x = \frac{2\ln 3}{\ln 5}$

(D) $x = \frac{\ln 3}{2\ln 5}$

$\ln 3^{2x} = \ln 5$

$2x \ln 3 = \ln 5$

$x = \frac{1}{2} \frac{\ln 5}{\ln 3}$

**Question 4**

If $a > 0$ and the function $f(x) = ax^3 + bx^2 + cx + d$ is always increasing, what is the condition for $a$, $b$ and $c$?

(A) $b^2 - 2ac > 0$

(B) $b^2 - 2ac < 0$

(C) $b^2 - 3ac > 0$

(D) $b^2 - 3ac < 0$

$f'(x) = 3ax^2 + 2bx + c$

If always increasing

$\Delta < 0$

$a > 0$

$\Delta = (2b)^2 - 4 \cdot 3a \cdot c < 0$

$4b^2 - 12ac < 0$

$b^2 - 3ac < 0$

**Question 5**

What is the number of solution(s) of the equation $log_e |x^2 - 1| = 0$ ?

(A) 2

(B) 3

(C) 1

(D) 0

$|x^2 - 1| = e^0 = 1$

$x^2 - 1 = 1$ or $x^2 - 1 = -1$

$x^2 = 2$ $\quad x^2 = 0$

$x = \pm\sqrt{2}$ $\quad x = 0$

*Examination continues next page…*

**Question 6**

Given that $f(x) = \dfrac{4x^5 - 8x}{x^3}$, what is the value of $f'(2)$?

(A) 2

(B) 8

(C) 12

(D) 18

$f(x) = 4x^2 - 8x^{-2}$

$f'(x) = 8x + 16x^{-3}$

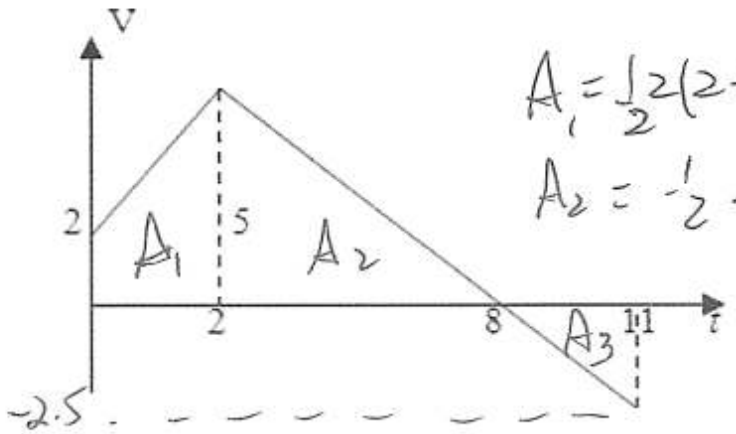
$f'(2) = 8(2) + 16(2)^{-3}$

$= 16 + 2$

$= 18$

**Question 7**

The diagram gives the velocity of a particle moving in a straight line for 11 seconds.




$A_1 = \frac{1}{2}2(2+5) = 7$

$A_2 = \frac{1}{2} \cdot 6.5 = 15$

$A_3 = \frac{1}{2} \times 3 \times 2.5$

$= 3.75$

At $t = 11$, the particle is

(A) 7 metres to the right of the starting point.

(B) 15 metres to the right of the starting point.

(C) 22 metres to the right of the starting point.

(D) 18.25 metres to the right of the starting point.

Distance travelled

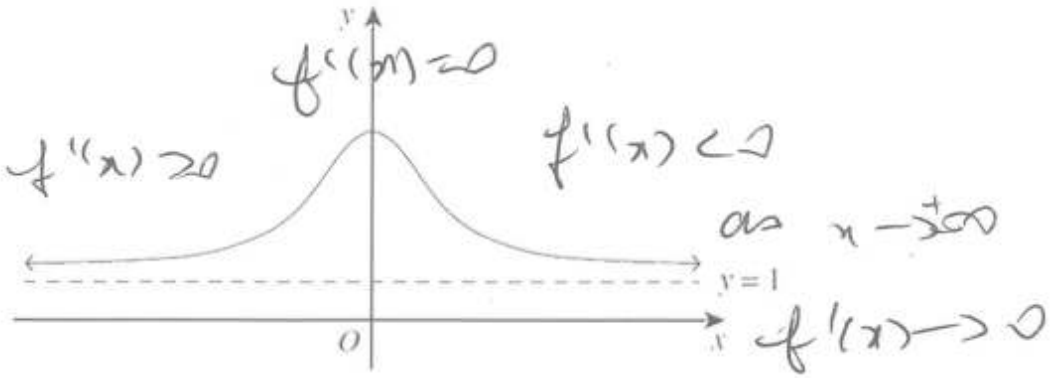
$= 7 + 15 - 3.75$

$= 18.25$ m

*Examination continues next page…*

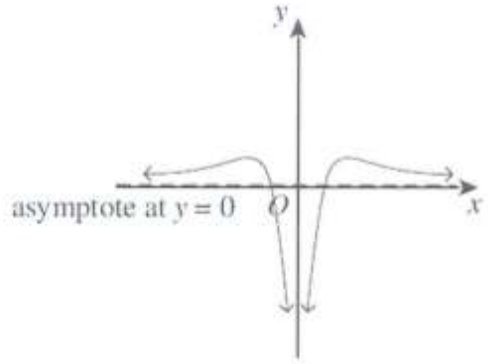
**Question 8**

The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = 1$.

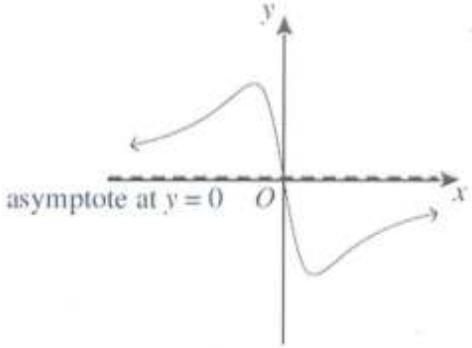



Which of the following could be the graph of $y = f'(x)$?

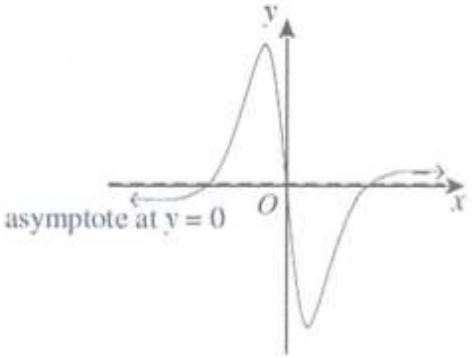
(A)



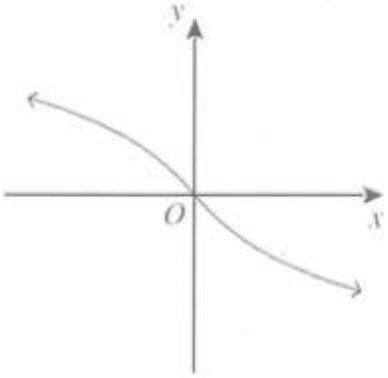

(B)




(C)




(D)




***Examination continues next page…***

**Question 9**

Given $\int_2^6 f(x)\ dx = 3$, what is the value of $\int_2^6 f(2(x-3)\ dx$?

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 3

(D) 8

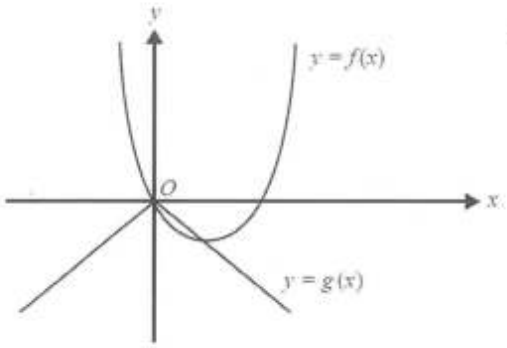
↑ horizontal dilation by a factor of $\frac{1}{2}$ area is halved

$\int_1^3 f(2x)\,dx = \frac{1}{2}\int_2^6 f(x)\,dx = \frac{3}{2}$

translating 3 units gives $\int_4^6 f[2(x-3)]\,dx = \frac{3}{2}$.

**Question 10**

The graph of $y = f(x)$ and $y = g(x)$ are shown below.

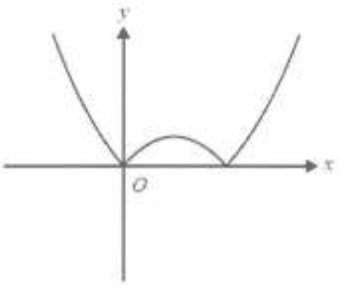



NOT TO SCALE

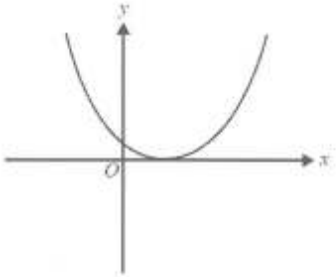
as $g(x) = 0$
$f(g(x)) = 0$
eliminate ⊗
$g(x) < 0$ for all $x$
and
$f(x) > 0$
when $x < 0$

The graph of $y = f(g(x))$ is best represented by

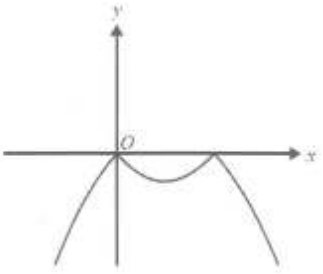
(A)



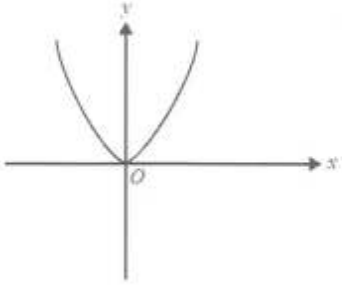

(B)




(C)




(D)




*Examination continues next page…*

## Section II

**90 marks**
**Attempt Questions 11–30**
**Allow about 2 hours and 45 minutes for this section**

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

---

**Question 11**

Solve $|4 - 2x| = 3$ 2

$4 - 2x = 3$ or $4 - 2x = -3$ (1)

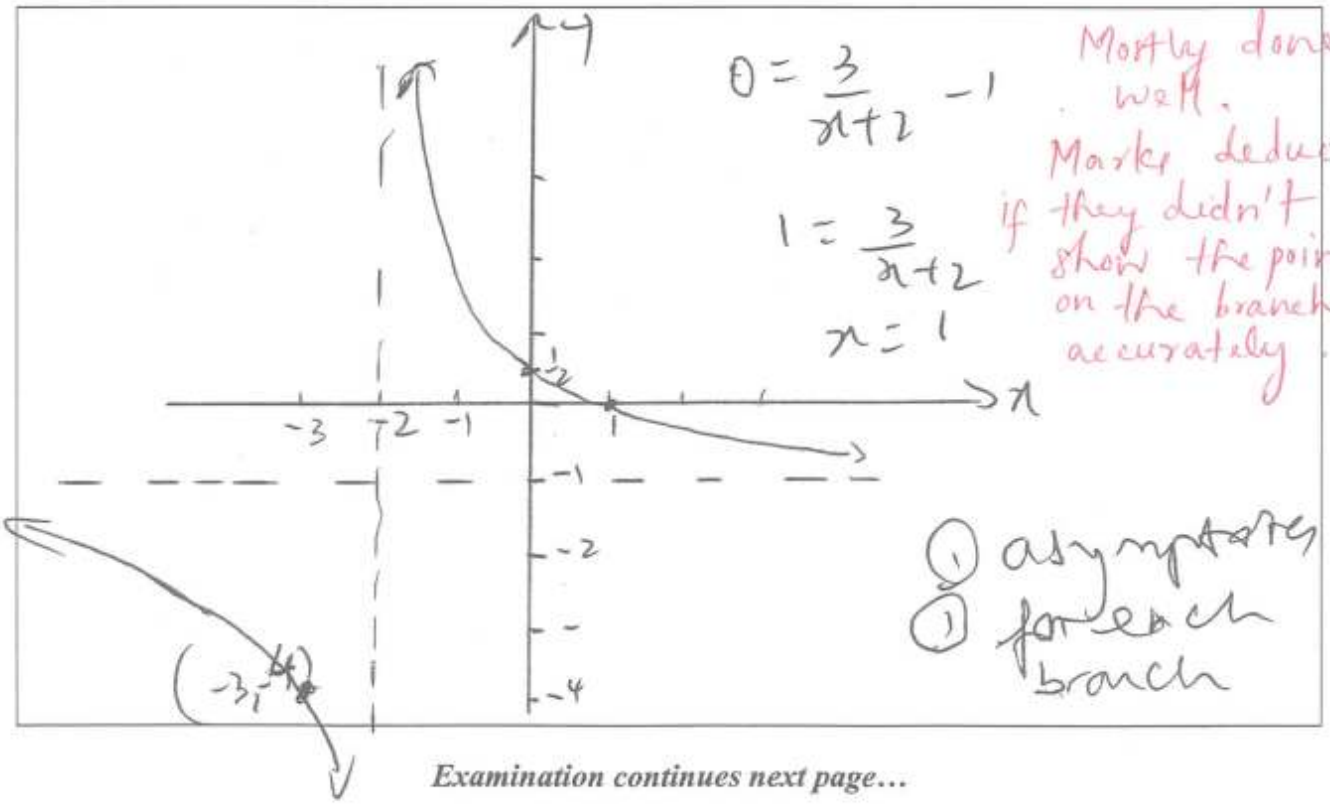
$-2x = -1$ $-2x = -7$

$x = \frac{1}{2}$ $x = \frac{7}{2}$ (1)

Well done.

**Question 12**

Sketch $y = \frac{3}{x+2} - 1$, showing all intercepts with the coordinate axes and all asymptotes. 3




*Examination continues next page…*

**Question 13**

Differentiate with respect to $x$:

a) $y = (3x^2 + 7)^6$ 2

$\frac{dy}{dx} = 6(3x^2 + 7)^5 \, 6x$ (1)

$= 36x(3x^2 + 7)^5$ (1)

Done well.

b) $y = 3x^2 \sin x$ 2

$\frac{dy}{dx} = 6x \cdot \sin x + \cos x \cdot 3x^2$

(1) (1)

Done well.

**Question 14**

a) $\int \frac{e^{2x}+1}{e^{2x}} \, dx$ 2

$= \int \frac{e^{2x}}{e^{2x}} + \frac{1}{e^{2x}} \, dx$

$= \int 1 + e^{-2x} \, dx$ (1)

$= x - \frac{1}{2}e^{-2x} + C$ (1)

Mostly done well. Some students made careless mistakes when integrating $e^{-2x}$.

*Examination continues next page…*

b) Evaluate $\int_0^2 x\sqrt{2x^2+1}\,dx$ /2

This is in the form

$$\int f'(x)\,f(x)^{n}\,dx$$

$$\frac{1}{4}\int_0^2 4x\sqrt{2x^2+1}\,dx$$

$$= \frac{1}{4}\left[\frac{2}{3}\left(2x^2+1\right)^{3/2}\right]_0^2 \quad \cdots$$

$$= \frac{1}{6}\left[\left(2x^2+1\right)^{3/2}\right]_0^2 \quad (1)$$

$$= \frac{1}{6}\left[\left(2.2^2+1\right)^{3/2} - \left(0+1\right)^{3/2}\right]$$

$$= \frac{1}{6}\left(27-1\right)$$

*Examination continues next page...*

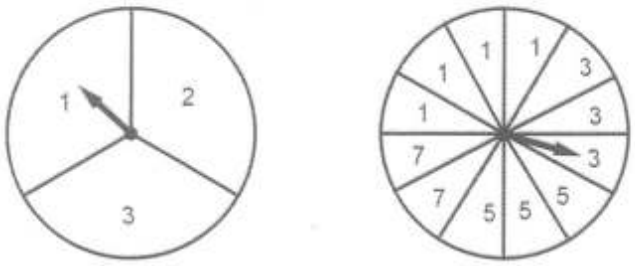
$$= \frac{13}{3} \quad (1)$$

Some students didn't use the formula correctly.

**Question 15**

The diagram below shows two spinners. 2

1st Spinner 2nd Spinner




Each of the three outcomes on the first spinner are equally likely.

On the second spinner there are 12 equally likely sectors for the arrow to land on with four possible outcomes.

In a game, both spinners are spun simultaneously.

The players score is the sum of the two numbers that the spinners land on.

In the diagram above, the player's score would be 4 since the first spinner landed on 1 and the second spinner landed on 3.

A player wins if their score is an odd number greater than 6.

What is the probability that a player will win on a single turn?

P(odd and > 6) = P(7 or 9)

Sample space (2,5) (2,7)

$= \frac{1}{3} \times \frac{3}{12} + \frac{1}{3} \times \frac{2}{12}$

$= \frac{5}{36}$

① partially correct

① correct answer

Mostly done well.

*Examination continues next page…*

**Question 16**

Jim tosses 2 dice with faces numbered 1 to 6. He records the maximum of the two upper faces as a score. The sample space is shown below.

| Dice 2 \ Dice 1 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 2 | 3 | 4 | 5 | 6 |
| 3 | 3 | 3 | 3 | 4 | 5 | 6 |
| 4 | 4 | 4 | 4 | 4 | 5 | 6 |
| 5 | 5 | 5 | 5 | 5 | 5 | 6 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

a) Jim plays a game with the 2 dice using the results stated above. 2

He wins \$1 if the score is 3 or 4, \$2 if the score is 6 and \$3 if it is 5.

But he loses \$1 if it is 1 or 2.

Done well.

Complete the probability distribution table for random variable $Y$ showing the probability of winning and losing on each amount.

$P(1) = \frac{1}{36}$ $P(2) = \frac{3}{36}$ $P(3) = \frac{5}{36}$ $P(4) = \frac{7}{36}$

$P(5)$

| $y$ | -\$1 | \$1 | \$2 | \$3 |
|---|---|---|---|---|
| ~~$P(Y=y)$~~ $P(y)$ | $\frac{4}{36}$ | $\frac{12}{36}$ | $\frac{11}{36}$ | $\frac{9}{36}$ |

① 2 correct
② all correct

b) Find the expected value of $Y$. 2

$E(Y) = 1 \times \frac{12}{36} + 2 \times \frac{11}{36} + 3 \times \frac{9}{36} - 1 \times \frac{4}{36}$

$= \$1.58$

① progress
② answer

Done well.

***Examination continues next page…***

c) If it costs Jim \$2 to play the game, would you expect him to win or lose money? 1

Justify your answer with calculations.

Loss $= -2 + 1.58$

$= -\$0.42$

Done well.

**Question 17**

For a particular curve it is given that $\frac{d^2y}{dx^2} = 12x - 2$. 3

The tangent at $(1, -2)$ on the curve makes an angle at $45^o$ with the positive direction of the $x$ axis.

Find the equation of the curve.

an angle of 45° means $\frac{dy}{dx} = 1$ (1)

$\frac{dy}{dx} = \int 12x - 2 \; dx$

To find $C_1$ some students substituted the point $(1, -2)$ instead of using $\frac{dy}{dx} = 1$ when $x = 1$.

$\frac{dy}{dx} = \frac{12x^2}{2} - 2x + C_1$

at $x = 1$ $\frac{dy}{dx} = 1$

$1 = 6(1)^2 - 2(1) + C_1 \quad \therefore \quad C_1 = -3$

$\frac{dy}{dx} = 6x^2 - 2x - 3$ (1)

$y = \int 6x^2 - 2x - 3 \; dx$

$y = 2x^3 - x^2 - 3x + C_2$

ents $(1, -2) \quad -2 = 2 - 1 - 3 + C_2$

$\therefore C_2 = 0$

$y = 2x^3 - x^2 - 3x$ (1)

*Examination continues next page…*

**Question 18**

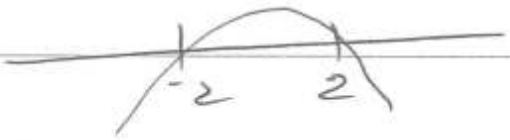
If $f(x) = \sqrt{x}$ and $g(x) = 4 - x^2$, what is the domain of the function $f(g(x))$? 2

$f(g(x)) = \sqrt{4 - x^2}$ (1)

solving quad inequalities was poor

$4 - x^2 \geq 0$

$(2 - x)(2 + x) \geq 0$




$-2 \leq x \leq 2$ (1)

**Question 19**

The point A (1, 4) lies on the graph $y = f(x)$. When the function is transformed into the graph $y = -2f(x + 1)$, what are the co-ordinates of the point corresponding to point A? 2

↑ vertical dilation & reflection

A (1, 4) → A (1, -8)

$f(x+1)$ is a horizontal translation

A (1, -8) → A (0, -8) to the left

(1) (1)

Common error (0, 8)

*Examination continues next page…*

**Question 20**

The energy released by an earthquake, $E$, can be given by $\log_{10} E = 11.8 + 1.5M$ where M is the measurement of its magnitude on the Richter scale.

a) Calculate the energy released by an earthquake measuring 6 on the Richter scale. 2
Leave your answer in index form.

$\log_{10} E_1 = 11.8 + 1.5 \times 6 = 20.8$ (1)

$E_1 = 10^{20.8}$ (1)

Common error $e^{20.8}$ not recognising it is base 10. well done

b) If there is another earthquake measuring 3.1 on the Richter scale, how many times more energy is released by the first earthquake than by the second? 2
Answer to the nearest whole number.

$\log_{10} E_2 = 11.8 + 1.5 \times 3.1$

$= 16.45$

$E_2 = 10^{16.45}$ (1)

$\frac{E_1}{E_2} = \frac{10^{20.8}}{10^{16.45}}$

$= 10^{4.35}$

$= 22387$ (nearest whole number) (1)

well done

*Examination continues next page…*

**Question 21**

The closure of several manufacturing businesses in a small town has caused the town's population, $P$, to decline over the last decade. At the start of 2012 the population was 15 000. At the start of 2022 it was estimated to be 9 500.

Assume that the population decline is proportional to $P$, so that $\frac{dP}{dt} = -kP$, where $k$ is a positive constant and $t$ is the time in years.

a) Show that $P = 15000e^{-kt}$ satisfies the differential equation. 1

$LHS = \frac{dP}{dt} = -k\,15000e^{-kt}$

$= -kP$

$= RHS$

well done.

b) Find the value of $k$ in exact form. 2

$9500 = 15000e^{-k\times 10}$

$\ln\frac{9500}{15000} = \ln e^{-10k}$ (1)

$\ln\frac{19}{30} = -10k \ln e$

$k = -\frac{1}{10}\ln\frac{19}{30}$ (1)

well done.

*Examination continues next page…*

c) If the population continues to decrease at the same rate, in what calendar year will the population drop to 5 000? 2

$5000 = 15000\, e^{-kt}$

$\ln \frac{1}{3} = \ln e^{-kt}$

$\ln \frac{1}{3} = -kt \ln e$

$t = \frac{-1}{k} \ln \frac{1}{3}$ (1)

$= \frac{-1}{-\frac{1}{10} \ln \frac{9}{30}} \ln \frac{1}{3}$

$= 24.05\ldots$

The population will drop to 5000 in the year 2036. (1)

Well done

common error: 2037.

*Examination continues next page…*

**Question 22**

A particle starts to move from the origin with a velocity 2 m/s along the $x$-axis.

Its acceleration at any time $t$ seconds is given by $a = \frac{6}{3t+4}$.

a) Show that the velocity, $v$, of the particle at time $t$ is 2

$$v = 2\ln\left(\frac{3}{4}t + 1\right) + 2 \quad m/s$$

$v = \int \frac{6}{3t+4}\,dt$

$= 2\int \frac{3}{3t+4}\,dt$ (1)

$v = 2\ln(3t+4) + c$

at $t = 0$, $v = 2$

$2 = 2\ln(3(0)+4) + c$

$c = 2 - 2\ln 4$

$v = 2\ln(3t+4) + (2 - 2\ln 4)$ (1)

$= 2\ln\left(\frac{3t+4}{4}\right) + 2$

$v = 2\ln\left(\frac{3t}{4} + 1\right) + 2$

well done up to here but students tried to fudge to get to $v = 2\ln\left(\frac{3}{4}t+4\right) + c$.

Poorly done.

b) Find the time taken by the particle to reach a velocity $4m/s$. 2

$V = 4$ $\quad 4 = 2\ln\left(\frac{3t}{4} + 1\right) + 2$ (1)

$2 = 2\ln\left(\frac{3t}{4} + 1\right)$

$1 = \ln\left(\frac{3t}{4} + 1\right)$

$e = \frac{3t}{4} + 1$

$t = \frac{4}{3}(e - 1)$ (1)

$t = 2.29$ sec

well done.

c) Does the particle ever return to the origin? Justify your answer. 1

Since velocity is always positive for $t \geq 0$, then velocity will increase in the positive direction.

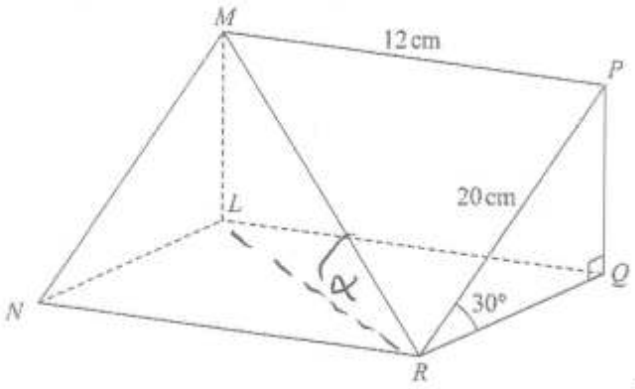
Hence the particle will never return to the origin.

well done.

*Examination continues next page…*

**Question 23**

The diagram shows a triangular prism. 3




Not to scale

Triangle PQR is a cross section of the prism.

$PR = 20cm,\ MP = 12cm,\ \angle PRQ = 30^o,\ \angle PQR = 90^o$

Calculate the size of the angle that the line $MR$ makes with the plane $RQLN$. Give your answer correct to 1 decimal place.

$\therefore \sin 30 = \frac{PQ}{20}$

$PQ = 20 \times \sin 30 = 10\ cm$ (1)

$MR = \sqrt{12^2 + 20^2}$ pythagoras Theorem

$= \sqrt{544}$ (1)

In $\triangle MLR$

$\sin \alpha = \frac{10}{\sqrt{544}}$ mixed results. Some students tried to find $\angle MRN$

$\alpha = 25.388...$

$\alpha = 25.4°$ (1)

*Examination continues next page…*

**Question 24**

The third term of a geometric series is 48 and the sixth term is ~~81/4~~ $\frac{81}{4}$.

a) Find the common ratio and the first term. 2

$T_3 = ar^2 = 48$ (1)

$T_6 = ar^5 = \frac{81}{4}$ (2)

(2) ÷ (1) $r^3 = \frac{27}{64}$

$r = \frac{3}{4}$ (1)

$a\left(\frac{3}{4}\right)^2 = 48$ well done.

$a = \frac{256}{3}$ (1)

b) Find the sum of the first ten terms to 4 significant figures. 2

$S_{10} = \frac{256}{3}\left(\frac{1-\left(\frac{3}{4}\right)^{10}}{1-\frac{3}{4}}\right)$ (1)

$= 322.1$ (4 sig fig) (1)

well done.

*Examination continues next page…*

**Question 25**

Consider the function $y = (\ln x)^2$.

a) Find its stationary point and determine its nature. 3

$\frac{dy}{dx} = 2\ln x \cdot \frac{1}{x}$ (1)

For stationary points $\frac{dy}{dx} = 0$

$\frac{2\ln x}{x} = 0$

$x = 1 \rightarrow y = 0$

stationary point at $(1,0)$ (1)

$\frac{d^2y}{dx^2} = 2 \cdot \frac{1}{x} \cdot \frac{1}{x} + -\frac{1}{x^2} \cdot 2\ln x$

$= \frac{2 - 2\ln x}{x^2}$

at $x = 1$

$\frac{dy}{dx^2} = \frac{2 - 2\ln 1}{1} = 2 > 0$ concave up

$\therefore (1,0)$ is a minimum turning point. (1)

Done well by most students

OR

| $x$ | 0.5 | 1 | 1.5 |
|---|---|---|---|
| $\frac{dy}{dx}$ | -2.8 | 0 | 0.54 |

\ - /

*Examination continues next page…*

b) Show that there is a point of inflexion at $(e, 1)$. 2

For point of inflexion $\frac{d^2y}{dx^2} = 0$

$$\frac{2 - 2\ln x}{x^2} = 0$$

$$2 = 2\ln x$$

$$\ln x = 1$$

$$x = e \rightarrow y = 1$$

(1)

| $x$ | 2 | $e$ | 3 |
|---|---|---|---|
| $\frac{d^2y}{dx^2}$ | 0.15 | 0 | −0.02 |

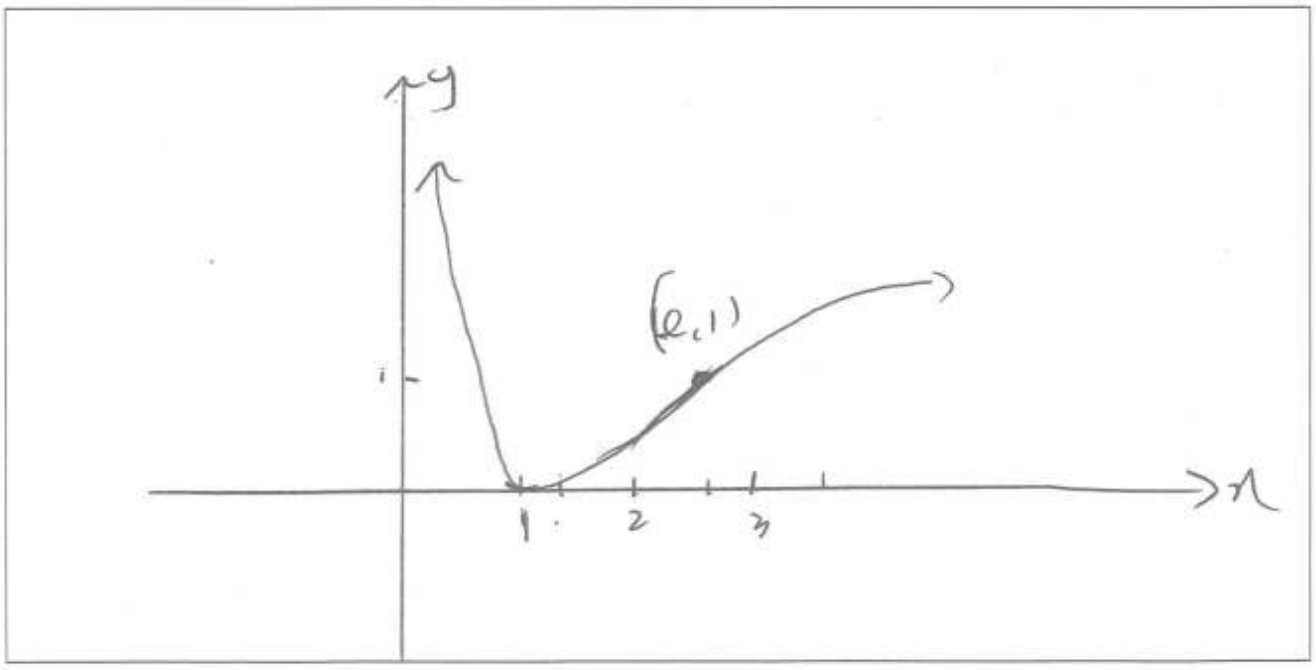
- Some students did not include values of $\frac{d^2y}{dx^2}$ in table ⇒ math test

(1)

- A small number of students did not test for change in concavity

Since there is a change in concavity $(e, 1)$ is a point of inflexion ✓

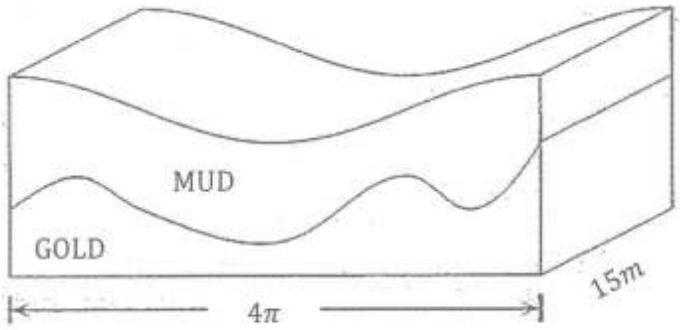
c) Sketch the function showing all essential features



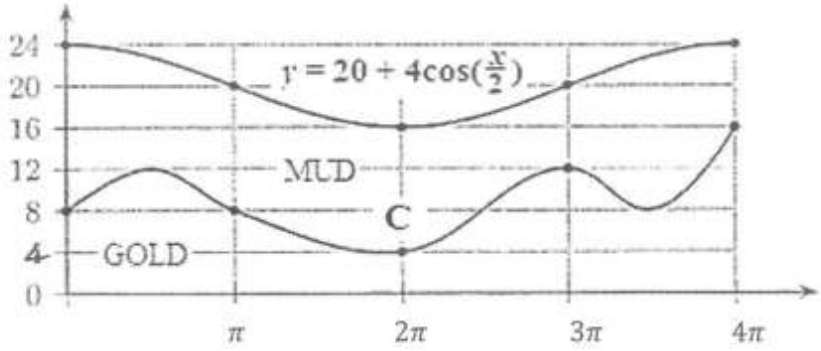

*Examination continues next page…*

**Question 26**

The diagram below shows an amount of gold ore, which is in the shape of a prism underneath a large amount of mud. The width of the prism is $4\pi$ metres and its length is 15 metres.




The graph below shows the cross-section of the prism. The top of the mud is given by the function $y = 20 + 4\cos\frac{x}{2}$ and the top of the gold is shown by the curve $C$.




a) Find, by integration, the total area of the cross-section of the prism, ie: the area of both the mud and gold. 2

$$A = \int_0^{4\pi} 20 + 4\cos\frac{x}{2}\,dx$$

$$= \left[20x + 2\times 4\times\sin\frac{x}{2}\right]_0^{4\pi} \quad (1)$$

$$= 20\times 4\pi + 8\sin\frac{4\pi}{2} - (0+0)$$

$$= 80\pi \text{ m}^2 \quad (1)$$

Done reasonably well.

b) Using the Trapezoidal Rule with the five function values on the graph, find an estimate for the area of the cross-section of the **gold**. 2

| $x$ | $0$ | $\pi$ | $2\pi$ | $3\pi$ | $4\pi$ |
|---|---|---|---|---|---|
| $y$ | $8$ | $8$ | $4$ | $12$ | $16$ |

(1) progress

$$A \doteq \frac{\pi}{2}\left[8 + 16 + 2(8 + 4 + 12)\right]$$

$$\doteq 36\pi$$ (1)

Done quite well. some miscalculated the value of h.

c) Find the volume of the mud. 1

$$V \doteq (80\pi - 36\pi) \times 15$$

$$\doteq 660\pi \text{ m}^3$$ (1)

Done reasonably well. - based on answers to a & b

*Examination continues next page…*

**Question 27**

The sum $S_n$ of the first $n$ terms of a certain series is $2n + 3n^2$, for $n \geq 1$. 3

Find an expression for the $n$th term $T_n$ of the series.

$S_{n-1} = 2(n-1) + 3(n-1)^2$

$= 2n - 2 + 3(n^2 - 2n + 1)$

$= 2n - 2 + 3n^2 - 6n + 3$

$= 3n^2 - 4n + 1$ (1)

$T_n = S_n - S_{n-1}$ (1)

$= 2n + 3n^2 - (3n^2 - 4(n+1)$

$= 2n + 3n^2 - 3n^2 + 4n - 1$

$T_n = 6n - 1$ (1)

A significant number of students failed to remember $T_n = S_n - S_{n-1}$

$\Rightarrow$ no marks awarded if working out did not lead them in the direction of the correct answer.

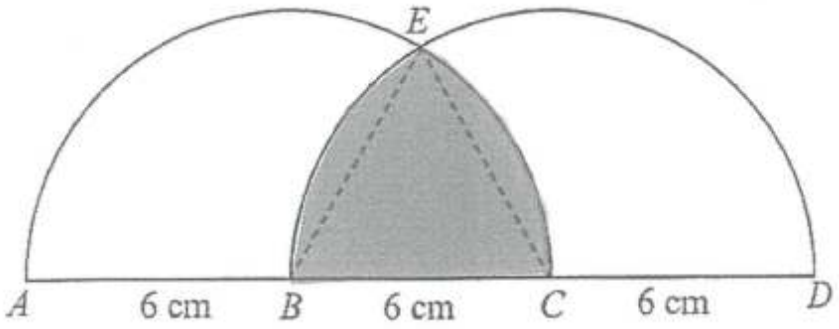
*Examination continues next page…*

**Question 28**

In the diagram, $ABCD$ is a straight line where $AB = BC = CD = 6cm$. 3

The semi-circles on diameter AC and BD intersect at $E$. $\Delta EBC$ is equilateral.

Find in simplest exact form the area of the shaded region common to the two semi-circles.




NOT TO SCALE

Since $\Delta EBC$ is equilateral

$\angle EBC = \angle ECB = \frac{\pi}{3}$

required area

= area of sector EBC in circle AEC

+ area of sector ECB in circle BED

- area $\Delta BCE$

$= \frac{1}{2} 6^2 \times \frac{\pi}{3} + \frac{1}{2} 6^2 \frac{\pi}{3} - \frac{1}{2} 6 \times 6 \sin \frac{\pi}{3}$ ① ①

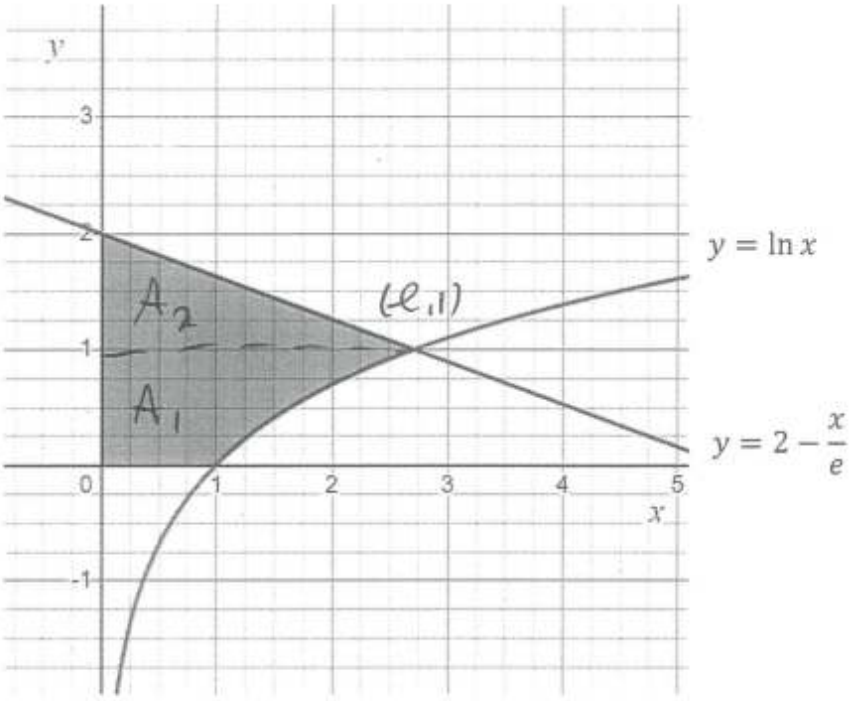
$= 12\pi - 9\sqrt{3}$ $cm^2$ ①

- marks lost mainly due to silly errors.

*Examination continues next page…*

**Question 29**

The shaded region below represents the area bounded by the $x$ and $y$ axes, the line $y = 2 - \frac{x}{e}$ and the curve $y = ln\, x$.




a) Show, by substitution, that the line $y = 2 - \frac{x}{e}$ and the curve $y = ln\, x$ intersect at the point $(e, 1)$. 1

Subs $(e,1)$ $y = 2 - \frac{x}{e}$

RHS $= 2 - \frac{e}{e} = 1 =$ LHS

Subs $(e,1)$ $y = \ln x$

RHS $= \ln e$

$= 1 =$ LHS

Students did well if following suggestion of substituting $x = e$ into equations.

*Examination continues next page…*

b) Hence, find the exact area of the shaded region. 3

$y = \ln x$

$x = e^y$

$A_1 = \int_0^1 e^y \, dy$ } ① properly

$= [e^y]_0^1$

$= e^1 - e^0$

$= e - 1$ ①

$A_2 = \frac{1}{2} \times 1 \times e = \frac{e}{2}$

$\text{total} = \frac{e}{2} + e - 1$ ①

*Examination continues next page...*

Done reasonably well.

Students attempting to rotate $y = \ln x$ around the $x$-axis ran into trouble.

**Question 30**

a) Use the change of base law to show the following is a geometric series and find the common ratio. 2

$$\log_3 x + \log_9 x + \log_{81} x + \cdots$$

$\frac{\log_e x}{\log_e 3} + \frac{\log_e x}{\log_e 9} + \frac{\log_e x}{\log_e 81} + \cdots$

$\frac{\log_e x}{\log_e 3} + \frac{\log_e x}{2\log_e 3} + \frac{\log_e x}{4\log_e 3}$ ① progression

$r = \frac{\log_e x}{2\log_e x} = \frac{\log_e x}{4\log_e 3} = \frac{1}{2}$ ①

$\frac{\log_e x}{\log_e 3}$ $\frac{\log_e x}{\log_e 3}$

*Working out was not set out properly

* students showed $\frac{T_2}{T_1} = \frac{1}{2}$ but did not show $\frac{T_3}{T_2} = \frac{1}{2}$, so it is a GP.

b) Hence, show that the sum to infinity of the series 2

$\log_3 x + \log_9 x + \log_{81} x + \cdots$ is $-\log_3 x^2$

Mostly students did not mention it.

Since $|r| < 1$ a limiting sum exists

$S = \frac{\log_3 x}{1 - \frac{1}{2}}$ ① $= 2\log_3 x$

$= \log_3 x^2$ ①

*Examination continues next page…*

**Question 31**

If $x = \frac{e^y - e^{-y}}{2}$ use the substitution $m = e^y$ to solve the equation for $y$ in terms of $x$. 4

$$x = \frac{m - \frac{1}{m}}{2} = \frac{m^2 - 1}{2m}$$

Majority got it wrong or could not do it.

$$2mx = m^2 - 1$$

$$m^2 - 2mx - 1 = 0 \quad (1)$$

$$m = \frac{-2x \pm \sqrt{(2x)^2 - 4.1.-1}}{2.1}$$

$$= \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$= \frac{2x \pm 2\sqrt{x^2 + 1}}{2}$$

$$m = x \pm \sqrt{x^2 + 1} \quad (1)$$

but $m = e^y$

$$e^y = x \pm \sqrt{x^2 + 1}$$

$e^y > 0$ ← students did not mention this

$$\therefore e^y = x + \sqrt{x^2 + 1} \quad (1)$$

$$\ln e^y = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$y = \ln\left|x + \sqrt{x^2 + 1}\right| \quad (1)$$

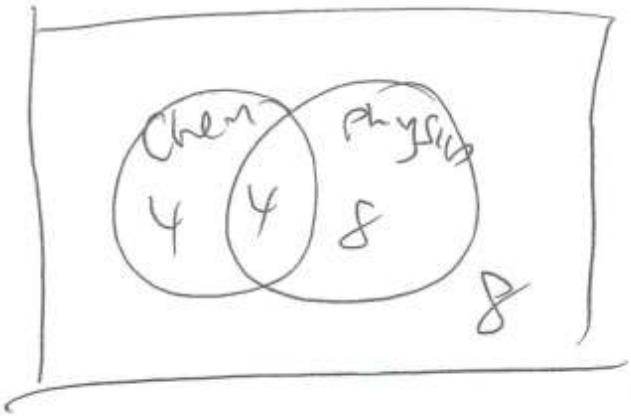
*Examination continues next page…*

**Question 32**

A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.
The results indicate that:

- 8 students chose to study neither Physics nor Chemistry
- 8 students chose to study Chemistry
- 12 students chose to study Physics.

a) Draw a Venn diagram showing this information. 1




mostly done well.

b) Let a student choosing to study Chemistry be event A. Let a student choosing to study Physics be event B. If a student is chosen at random, determine whether these events are independent of each other. 2

$P(A) = \frac{8}{24} = \frac{1}{3}$ $P(B) = \frac{12}{24} = \frac{1}{2}$ } 1

$P(A \cap B) = \frac{4}{24} = \frac{1}{6}$

Also can show $P(A|B) = P(A)$ and $P(B|A) = P(B)$

$P(A) \times P(B) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ progress

Since $P(A \cap B) = P(A) \times P(B)$ 1

The events A, B are independent.

Mostly done wrong. Students do not understand the concept.

*Examination continues next page…*

**Question 33**

The following functions represent 3

the number of daytime hours $D = 11.87 + 2.35\cos\frac{\pi}{183}(t + 10)$

and the number of night time hours $N = 12.13 - 2.35\cos\frac{\pi}{183}(t + 10)$

each day in Sydney during 2024, where $t = 0$ is 1st January, 2024.

Find the two values of $t$ when the number of hours of daytime and night time in Sydney are equal.

$11.87 + 2.35\cos\frac{\pi}{183}(t+10)$

$= 12.13 - 2.35\cos\frac{\pi}{183}(t+10)$

$4.7\cos\frac{\pi}{183}(t+10) = 0.26$ (1)

$\cos\frac{\pi}{183}(t+10) = 0.055\ldots$

$\frac{\pi}{183}(t+10) = 1.5154.., 2\pi - 1.5154..$

$= 1.5154..., 4.7677..$ (1)

$t = 78.275.., 267.72$ (1)

Some students well but majority could not find the correct values of $t$.

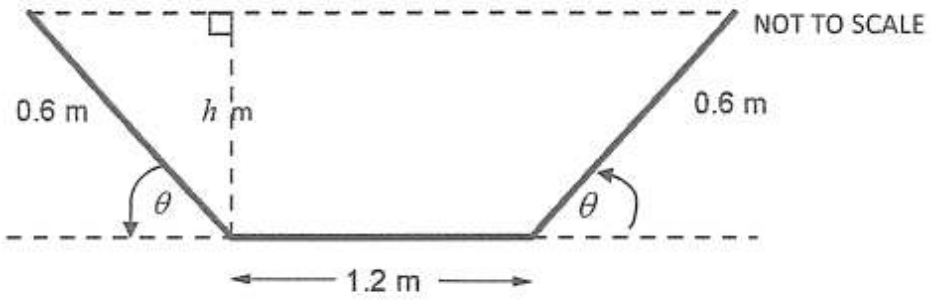
*Examination continues next page…*

**Question 34**

A water overflow channel is to be dug next to a dam. The base of the channel is to be 1.2 metres wide and 15 metres long.

The channel is dug such that the left and right sides are on an angle of $\theta$ with the horizontal. The cross-sectional view of the channel is a trapezium with two sides of length 0.6 m.

The diagram below shows a cross-sectional view of the channel.




a) Find an expression for $h$ in terms of $\theta$. 1

$\sin\theta = \frac{h}{0.6}$

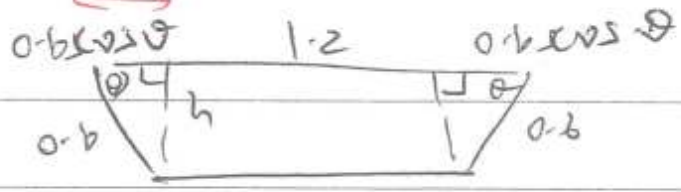
Well done.

$h = 0.6\sin\theta$

*Examination continues next page…*

b) Show that the volume of the channel is given by 2

$$V = 5.4(2\sin\theta + \sin\theta\cos\theta) \text{ cubic metres}$$




$$A = \frac{1}{2}h\left[1.2 + (1.2 + 0.6\cos\theta \times 2)\right]$$

$$= \frac{1}{2}h\left[2.4 + 1.2\cos\theta\right]$$ ①

Subs $h = 0.6\sin\theta$

$$= \frac{1}{2}\,0.6\sin\theta\left(2.4 + 1.2\cos\theta\right)$$

$$= 0.3\,6\sin\theta\,(2 + \cos\theta)$$

$V = AH$

$V = 0.36\sin\theta\,(2 + \cos\theta) \times 15$ ①

$= 5.4\,(2\sin\theta + \sin\theta\cos\theta)$

instead of $x = 0.6\cos\theta$

Mostly done well but some students used complicated expression using Pythagoras theorem.

*Examination continues next page…*

c) Find the value of the angle $\theta$ so that the volume of the channel is a **maximum**. 4

Give your answer correct to the nearest minute.

$V = 5.4(2\sin\theta + \sin\theta\cos\theta)$

$\frac{dV}{d\theta} = 5.4(2\cos\theta + \cos\theta\cos\theta - \sin\theta.\sin\theta)$

$= 5.4(2\cos\theta + \cos^2\theta - \sin^2\theta)$

$= 5.4(2\cos\theta + \cos^2\theta - (1-\cos^2\theta))$

$= 5.4(2\cos^2\theta + 2\cos\theta - 1) = 0$ (1)

For stationary points

$a = 2 \quad b = 2 \quad c = -1$

$\cos\theta = \frac{-2 \pm \sqrt{2^2 - 4.2.-1}}{2.2}$

$= \frac{-2 \pm \sqrt{12}}{4}$

$= \frac{-1 + \sqrt{3}}{2}$ (1)

$\cos\theta = \frac{-1-\sqrt{3}}{2}$ $\quad$ $\cos\theta = \frac{-1+\sqrt{3}}{2}$

no solution $\quad$ $\theta = 68°32$ (1)

$0 \leq \theta \leq 90°$

Some students got full marks, did well every step, others made errors

$$\frac{d^2V}{d\theta^2} = 5.4[2\cos\theta.\sin\theta + -2\sin\theta]$$

$$= 5.4(4\cos\theta\sin\theta - 2\sin\theta]$$

when $\theta = 68°32'$

$$\frac{d^2V}{d\theta^2} = 5.4(4\cos 68°32'\sin 68°32' - 2\sin 68°32'] \quad (1)$$

$< 0 \quad (-2.6944)$

(Concave down)

$\therefore$ V is maximum when $\theta = 68°32'$

Many students lost a mark because they did not show its a maxima

*End of examination.*